Nature of the energy transfer process in compressible turbulence

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Using a two-point closure theory, the eddy-damped-quasinormal-Markovian approximation, we have investigated the energy transfer process and triadic interactions of compressible turbulence. In order to analyze the compressible mode directly, the Helmholtz decomposition is used. The following issues were addressed: (1) What is the mechanism of energy exchange between the solenoidal and compressible modes, and (2) is there an energy cascade in the compressible energy transfer process? It is concluded that the compressible energy is transferred locally from the solenoidal part to the compressible part. It is also found that there is an energy cascade of the compressible mode for high turbulent Mach number. Since we assume that the compressibility is weak, the magnitude of the compressible (radiative or cascade) transfer is much smaller than that of the solenoidal cascade. These results are further confirmed by studying the triadic energy transfer function, the most fundamental building block of the energy transfer. [S1063-651X(99)10304-0]

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I. INTRODUCTION

It is well known that compressible turbulence plays a prominent role in a wide range of important scientific and engineering applications, including high speed transport, supersonic combustions, and acoustics. Recently, a large body of publications has been devoted to study various aspects of compressible turbulence using both direct numerical simulations (DNS) and large eddy simulations (LES). Here we simply mention a few representative works, such as Feiereisen, Reynolds, and Ferziger [1]; Passot and Pouquet [2]; Lee, Lele, and Moin [3]; Sarkar, Erlebacher, and Hussaini [4]; Erlebacher *et al.* [5]; Kida and Orszag [6]; Blaisdell, Mansour, and Reynolds [7]; and Porter, Pouquet, and Woodward [8]. For a comprehensive review, the reader is referred to Lele [9]. These numerical simulations have substantially improved our understanding of compressible turbulence. Nevertheless, some basic physical processes of compressible turbulence, such as the energy transfer and triadic interactions, have not been explored even at low turbulent Mach number. For example, do we expect an energy cascade process of compressible velocity modes? How does the energy exchange between the solenoidal and compressible modes take place? These types of studies require a substantial spectral scale range of interactions. As a result, it is very hard to utilize the DNS databases since these simulations are limited to very low Reynolds numbers and have only very limited spectral ranges. While LES can provide databases at substantially higher Reynolds numbers, subgrid models must be introduced.

Another way to generate high Reynolds number databases is by using two-point closure models. The direct interaction approximation (DIA) of Kraichnan [10] is a well-established approach. Many authors have studied incompressible turbulence using DIA (see Leslie [11]). The method of eddy-damped-quasinormal-Markovian (EDQNM) models (Orszag [12]) has been shown as a simpler, but effective, alternative to DIA. Recently, Bertoglio, Bataille, and Marion [13] have presented DIA and EDQNM equations for a weakly compressible turbulence.

The aim of this paper is to use the EDQNM closure theory to study the energy transfer and triadic interactions of compressible turbulence. The paper is organized as follows: First, we review closure assumptions of EDQNM and present the resulting transport equations. Second, we perform a detailed analysis of the nonlinear transfer terms. Finally, we investigate the most fundamental aspect of the energy transfer process, the incompressible and compressible triadic interactions.

II. COMPRESSIBLE EDQNM MODEL

The basic set of equations are the Navier-Stokes and continuity equations. The fluid is assumed to be homogeneous, isotropic, and barotropic. Reynolds average and Fourier transform are used to obtain the fluctuating turbulent field equations in spectral space. The equations are partially linearized with respect to the density fluctuation that leads to the condition $M_t<1$, where M_t is the turbulent Mach number defined as $M_t=\sqrt{q^2}/c_0$. q^2 is twice the turbulent kinetic energy, and c_0 is the sound speed. To analyze the compressibility effects, we use the Helmholtz decomposition to split the velocity vector into a solenoidal part $u^S(\mathbf{K},t)$, which corresponds to the velocity fluctuations perpendicular to the wave vector $\mathbf{K}$ in the Fourier space, and a compressible part $u^C(\mathbf{K},t)$, which corresponds to fluctuations in the direction of the wave vector.

The classical DIA approach (two point–two times) is used

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to derive compressible DIA equations. These equations are the starting point used to derive the compressible EDQNM equations (in two points and one time). The methodology is the same as that of the incompressible case [Leslie [11] and Lesieur [14]] but more equations are now needed [15].

A. Spectral equations

The EDQNM governing equations of weakly compressible turbulence are the following: (i) an equation for the spectrum (E^{SS}), the autocorrelation of the solenoidal part of the velocity field,

$$\frac{\partial}{\partial t} E^{SS}(K,t) = -2\nu K^2 E^{SS}(K,t) + T^{SS}(K,t); \tag{1}$$

(ii) an equation for the spectrum (E^{CC}), the autocorrelation of the ‘‘purely compressible’’ part of the velocity field,

$$\frac{\partial}{\partial t} E^{CC}(K,t) = -2\nu' K^2 E^{CC}(K,t) + T^{CC}(K,t) - 2c_0 K E^{CP}(K,t); \tag{2}$$

(iii) an equation for the spectrum of the potential energy (E^{PP}) associated with the pressure,

$$\frac{\partial}{\partial t} E^{PP}(K,t) = 2c_0 K E^{CP}(K,t); \tag{3}$$

(iv) an equation for the spectrum of the pressure-velocity correlation (E^{CP}),

$$\frac{\partial}{\partial t} E^{CP}(K,t) = -\nu' K^2 E^{CP}(K,t) + T^{CP}(K,t) + c_0 K\big(E^{CC}(K,t) - E^{PP}(K,t)\big). \tag{4}$$

In the case of a Stokesian fluid,

$$\nu' = \frac{\lambda + 2\mu}{\langle \rho \rangle} = \tfrac{4}{3}\nu, \tag{5}$$

μ and λ, two dynamic viscosities, are assumed to be uniform.

B. Energy transfer terms

In Eqs. (1)–(4), T^{SS}, T^{CC}, and T^{CP} are the transfer terms. They contain several contributions:

$$T^{SS} = T_1^{SS} + T_2^{SS} + T_3^{SS} + T_4^{SS} + T_5^{SS}, \tag{6}$$

$$T^{CC} = T_1^{CC} + T_2^{CC} + T_3^{CC} + T_4^{CC} + T_5^{CC} + T_6^{CC}, \tag{7}$$

$$T^{CP} = T_1^{CP} + T_2^{CP} + T_3^{CP} + T_4^{CP} + T_5^{CP} + T_6^{CP}. \tag{8}$$

Different contributions appearing in the transfer term acting on the solenoidal field are

$$T_1^{SS} = \int_\Delta \frac{K^3}{PQ} \frac{1 - xyz - 2y^2z^2}{2} \theta_{KPQ}^{SS-SS-SS} \times E^{SS}(P,t) E^{SS}(Q,t)\, dP\, dQ, \tag{9}$$

$$T_2^{SS} = \int_\Delta \frac{K^3}{PQ} \frac{(1-y^2)(x^2+y^2)}{1-x^2} \theta_{KPQ}^{SS-SS-CC} \times E^{SS}(P,t) E^{CC}(Q,t)\, dP\, dQ, \tag{10}$$

$$T_3^{SS} = -\int_\Delta \frac{P^2}{Q} (xy + z^3)\, \theta_{KPQ}^{SS-SS-SS} E^{SS}(K,t) E^{SS}(Q,t)\, dP\, dQ, \tag{11}$$

$$T_4^{SS} = \int_\Delta \frac{P^2}{Q} (2xy)\, \theta_{KPQ}^{SS-SS-CC} E^{SS}(K,t) E^{CC}(Q,t)\, dP\, dQ, \tag{12}$$

$$T_5^{SS} = -\int_\Delta \frac{P^2}{Q} \big(z(1-z^2)\big) \theta_{KPQ}^{SS-CC-SS} \times E^{SS}(K,t) E^{SS}(Q,t)\, dP\, dQ. \tag{13}$$

Different contributions to the transfer term in the E^{CC} equation are

$$T_1^{CC}(K,t) = \int_\Delta \frac{K^3}{PQ} \big((x+yz)^2\big) \theta_{KPQ}^{CC-SS-SS} \times E^{SS}(P,t) E^{SS}(Q,t)\, dP\, dQ, \tag{14}$$

$$T_2^{CC}(K,t) = \int_\Delta \frac{K^3}{PQ} \frac{(x^2-y^2)^2}{(1-x^2)} \theta_{KPQ}^{CC-SS-CC} \times E^{SS}(P,t) E^{CC}(Q,t)\, dP\, dQ, \tag{15}$$

$$T_3^{CC}(K,t) = \int_\Delta \frac{K^3}{PQ} (x^2)\, \theta_{KPQ}^{CC-CC-CC} \times E^{CC}(P,t) E^{CC}(Q,t)\, dP\, dQ, \tag{16}$$

$$T_4^{CC}(K,t)=-\int_\Delta \frac{P^2}{Q} 2z(1-z^2)\,\theta_{KPQ}^{CC-SS-SS} \times E^{CC}(K,t)E^{SS}(Q,t)dPdQ, \tag{17}$$

$$T_5^{CC}(K,t)=-\int_\Delta \frac{P^2}{Q}(2z^3-z+xy)\,\theta_{KPQ}^{CC-CC-SS} \times E^{SS}(Q,t)E^{CC}(K,t)dPdQ, \tag{18}$$

$$T_6^{CC}(K,t)=\int_\Delta \frac{P^2}{Q}(2xy)\,\theta_{KPQ}^{CC-CC-CC} \times E^{CC}(K,t)E^{CC}(Q,t)dPdQ. \tag{19}$$

And finally, the contributions to the transfer term in the pressure velocity correlation are

$$T_1^{CP}(K,t)=\int_\Delta \frac{K^3}{PQ}\frac{(x+yz)^2}{2}\,\theta_{KPQ}^{PC-SS-SS} \times E^{SS}(P,t)E^{SS}(Q,t)dPdQ, \tag{20}$$

$$T_2^{CP}(K,t)=\int_\Delta \frac{K^3}{PQ}\frac{(x^2-y^2)^2}{2(1-x^2)}\,\theta_{KPQ}^{PC-SS-CC} \times E^{SS}(P,t)E^{CC}(Q,t)dPdQ, \tag{21}$$

$$T_3^{CP}(K,t)=\int_\Delta \frac{K^3}{PQ}\frac{x^2}{2}\,\theta_{KPQ}^{PC-CC-CC} \times E^{CC}(P,t)E^{CC}(Q,t)dPdQ, \tag{22}$$

$$T_4^{CP}(K,t)=-\int_\Delta \frac{P^2}{Q} z(1-z^2)\,\theta_{KPQ}^{PC-SS-SS} \times E^{CC}(K,t)E^{SS}(Q,t)dPdQ, \tag{23}$$

$$T_5^{CP}(K,t)=-\int_\Delta \frac{P^2}{Q}\frac{2z^3-z+xy}{2}\,\theta_{KPQ}^{PC-CC-SS} \times E^{SS}(Q,t)E^{CC}(K,t)dPdQ, \tag{24}$$

$$T_6^{CP}(K,t)=\int_\Delta \frac{P^2}{Q}(xy)\,\theta_{KPQ}^{PC-CC-CC} \times E^{CC}(K,t)E^{CC}(Q,t)dPdQ. \tag{25}$$

The integration in the (P,Q) plane extends over a domain such that **K**, **P**, and **Q** form a triangle. The expressions x,y,z are standard coefficients associated with the geometry of the triad and are the cosines of the angles, respectively, opposite to **K**, **P**, **Q**. Furthermore, temporal functions (defined by θ's) are the decorrelation times deduced by integrating the DIA equations over time.

The transfer terms appearing at the transport equations for the solenoidal and compressible modes will be analyzed in detail in the next section. Since there is no transfer term in the equation for the pressure-pressure correlation, this equation characterizes the exchange between the solenoidal mode and the compressible mode ($E^{CC}+E^{PP}$). Finally, the effect of T^{CP} is to interchange energy between E^{CC} and E^{PP} [16].

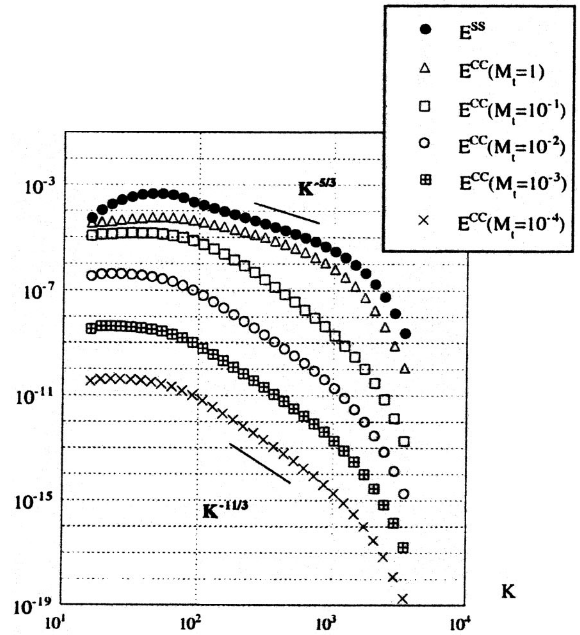

FIG. 1. Solenoidal and compressible velocity correlations.

C. Energy spectra

A force is applied in the large scales of the solenoidal velocity. This forcing is only used on the solenoidal mode in order to not perturbate the evolution of the compressible mode. It is realized by frozening the values of the solenoidal spectrum in the small wave-number range (until $K=64m^{-1}$). Due to this forcing, small discontinuities can appear on the solenoidal spectrum around this value of K but they have no influence on the results. Our analysis is carried out when both solenoidal and compressible modes have reached their asymptotic stationary states.

In Fig. 1, we present the spectra of both solenoidal and compressible components of the velocity correlation corresponding after the asymptotic state is reached. The Taylor microscale Reynolds number, Re, is approximately 140. Here $\mathrm{Re}\equiv(q^2/3)\sqrt{15/\nu\epsilon}$ and ϵ is the dissipation. In the figures, the dimensional unit of the correlation spectra is given as $\mathrm{m}^3\,\mathrm{s}^{-2}$ and the dimensional unit of wave number K is defined as m^{-1}. The energy associated with the purely compressible mode is found to vary as the square of the turbulent Mach number. At low and moderate M_t, the spectrum of the compressible component shows a $K^{-11/3}$ behavior in the inertial range [16,17].

In order to examine the dependency of the turbulent Mach number, it was allowed to vary from 10^{-4} to 1. Note, however, that, strictly speaking, our model is valid only for small

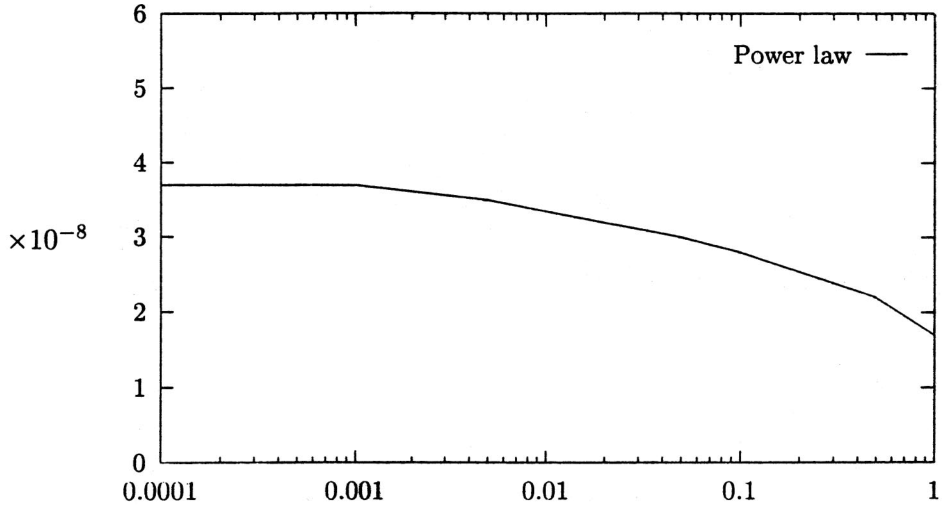

FIG. 2. Influence of the turbulent Mach number on the compressible power law.

M_t. Nevertheless, computations were carried out up to $M_t = 1$ in order to study the limiting behavior of the model. Furthermore, working with this value ($M_t = 1$) has the advantage that we can better visualise the results obtained with smaller values.

Figure 2 gives the power law in the inertial range. We can observe the behavior in $K^{-11/3}$ for turbulent Mach numbers less than 0.01. When the turbulent Mach number increases, the shape of the compressible spectrum evolves and becomes close of $K^{-5/3}$ for $M_t = 1$. We will see later that a compressible energy cascade is responsible of the slope reduction.

III. STUDY OF THE TRANSFER TERMS

A. Solenoidal transfer term

In Fig. 3, we observe that T^{SS} has the usual shape observed in incompressible turbulence studies. Specifically, T^{SS} is negative in the large scales and positive for small scales. Physically this corresponds to the energy transfer from the large scales to the smaller ones. The transfer terms are given in $m^3\,s^{-3}$.

There are 17 contributors to the transfer function of compressible turbulence. This should be be compared with its counterpart of incompressible turbulence:

$$T_s^{SS}(K,t) = T_1^{SS}(K,t) + T_3^{SS}(K,t). \tag{26}$$

Indeed, the two key contributors to T^{SS} are T_1^{SS} and T_3^{SS}, which are the same terms that one finds in incompressible turbulence. These terms are usually called ''input'' and ''output'' terms. For our weakly compressible turbulence, they are much more important contributors than the others (see Fig. 3). Indeed, other contributors are negligible in comparison with T_1^{SS} and T_3^{SS}. We stress that the summation of these two terms is the net energy transfer of incompressible

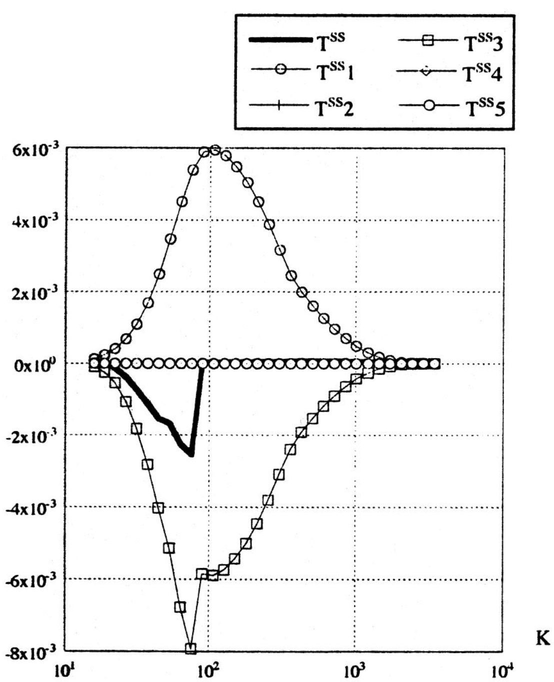

FIG. 3. Contributions of the solenoidal transfer term.

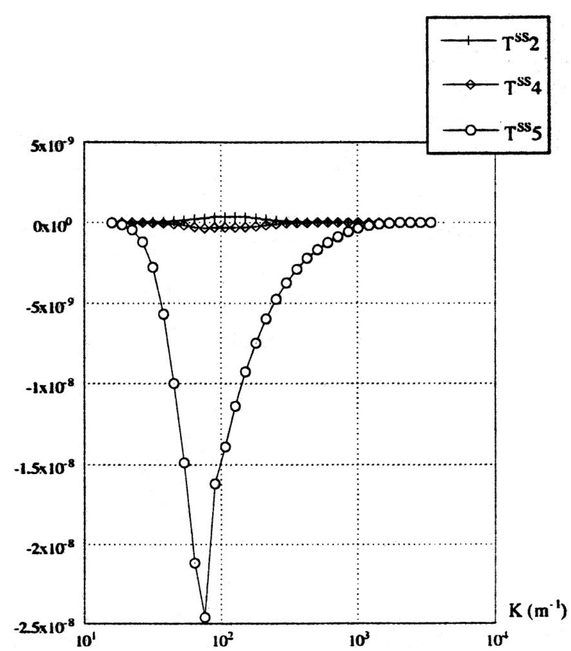

FIG. 4. Contributions of the solenoidal transfer term.

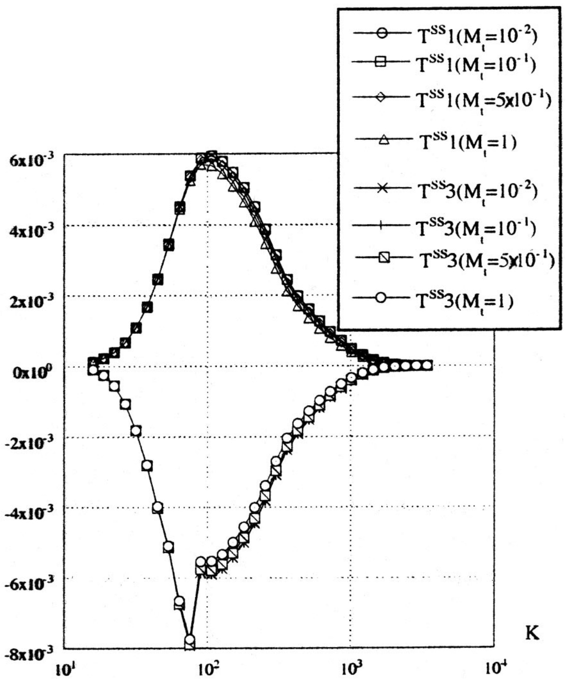

FIG. 5. T_1^{SS} and T_3^{SS} at different M_t.

turbulence (solid line). To clearly illustrate the behaviors of the new terms, we plotted these ‘‘compressible’’ contributors (T_2^{SS}, T_4^{SS}, and T_5^{SS}) in an enlarged scale (Fig. 4) in order to observe their relative magnitudes. We found that T_5^{SS} is much larger than T_2^{SS} and T_4^{SS}, and consequently it makes the largest contribution to the compressible effects in T^{SS}. An important feature of this term is that it is negative for all spectral space, indicating that the energy transfer has been transferred from the solenoidal mode to the compressible mode.

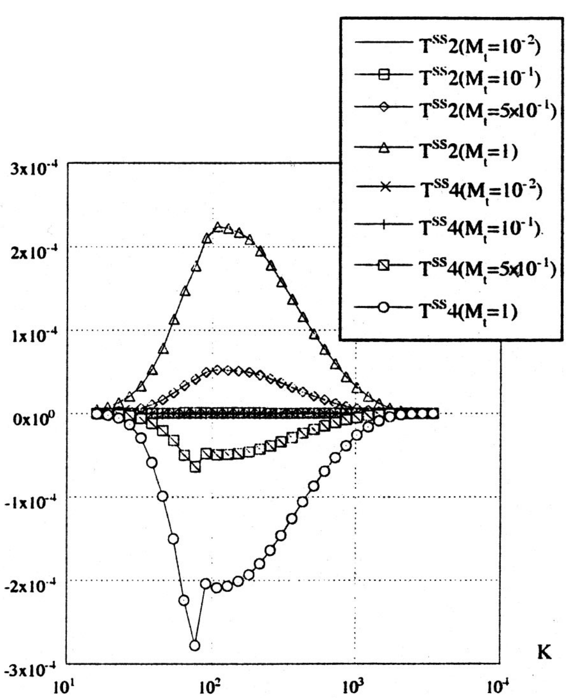

FIG. 6. T_2^{SS} and T_4^{SS} at different M_t.

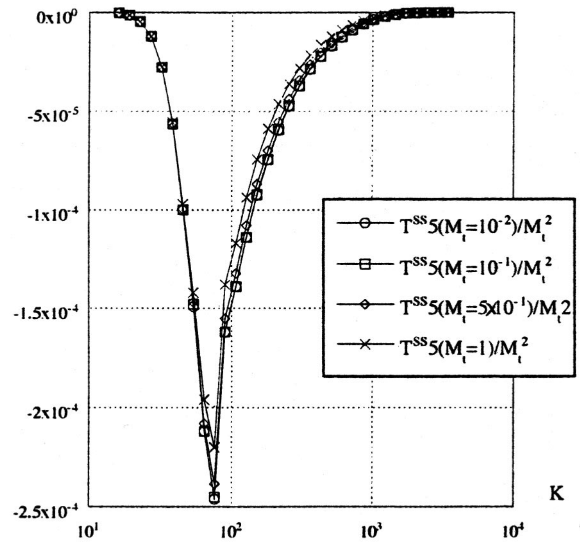

FIG. 7. T_5^{SS} divided by M_t^2.

We found that the compressibility has negligible effect on the solenoidal transfer term T^{SS}. The reason is that the dominant terms in T^{SS}, e.g., the incompressible contributions (T_1^{SS} and T_3^{SS}), are independent of M_t (Fig. 5). On the other hand, the ‘‘compressible’’ contributions (which are smaller) depend strongly on the values of the turbulent Mach number. Figure 6 illustrates the dependencies of T_2^{SS} and T_4^{SS} on the turbulent Mach number. We found that all the spectra of T_5^{SS} can be collapsed by dividing M_t^2 (Fig. 7). Therefore, we conclude that T_5^{SS} is proportional to M_t^2, a result that can be found analytically. Since the dominant contributions in T^{SS} are insensitive to the variations of turbulent Mach number, T^{SS} is not affected by the compressibility.

B. Compressible transfer term

In this subsection, we will study the transfer term T^{CC}, and its individual contributors. This compressible transfer

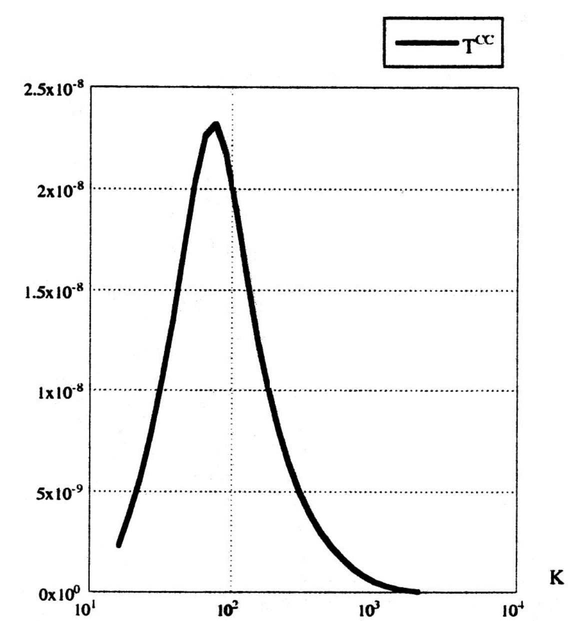

FIG. 8. Compressible transfer term for $M_t=0.01$.

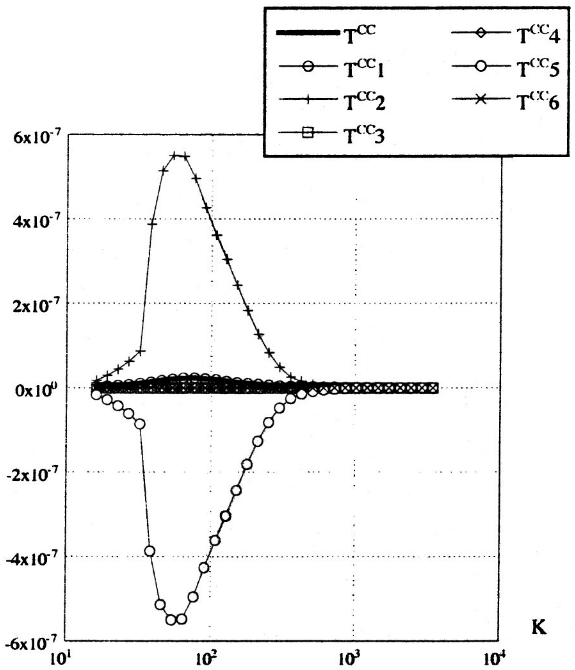

FIG. 9. Contributions of the compressible transfer term.

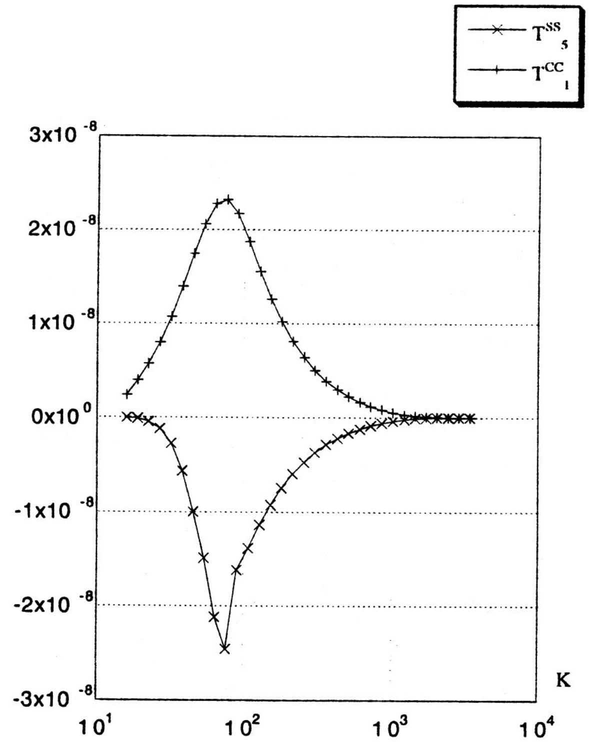

FIG. 11. T_5^{SS} and T_1^{CC}.

term appears in the transport equation of the compressible autocorrelation. In Fig. 8, the compressible transfer term is plotted at $M_t = 10^{-2}$, we can observe that the compressible transfer is positive for all spectral space. Hence, it is a term that is responsible for the production of compressible energy. It has to be note that, for this turbulent Mach number, the maximum of magnitude of the compressible transfer term is down by five orders of magnitude from the maximum of the solenoidal transfer term. The order of magnitude of the compressible energy transfer changes with the turbulent Mach number. Nevertheless, the incompressible transfer term is always bigger than the compressible transfer term.

The different contributors to T^{CC} are plotted in Fig. 9. It is clear that the two terms, T_2^{CC} and T_5^{CC}, are much larger than the others. Another term, T_1^{CC} is a distant third in size. To illustrate the relative size of the smaller terms, we replotted these contributors in Fig. 10 at the enlarged scale. Because two dominant terms have similar magnitudes but opposite signs, a strong cancellation between them is expected. Indeed for all cases considered, the term T_2^{CC} is always posi-

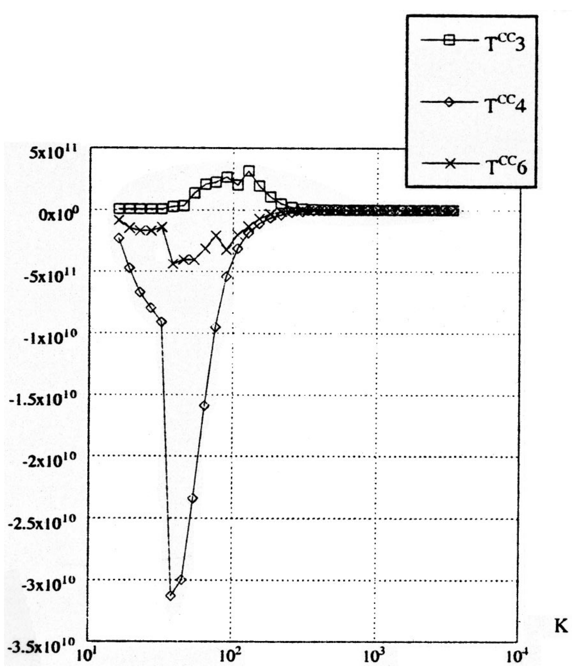

FIG. 10. Contributions of the compressible transfer term.

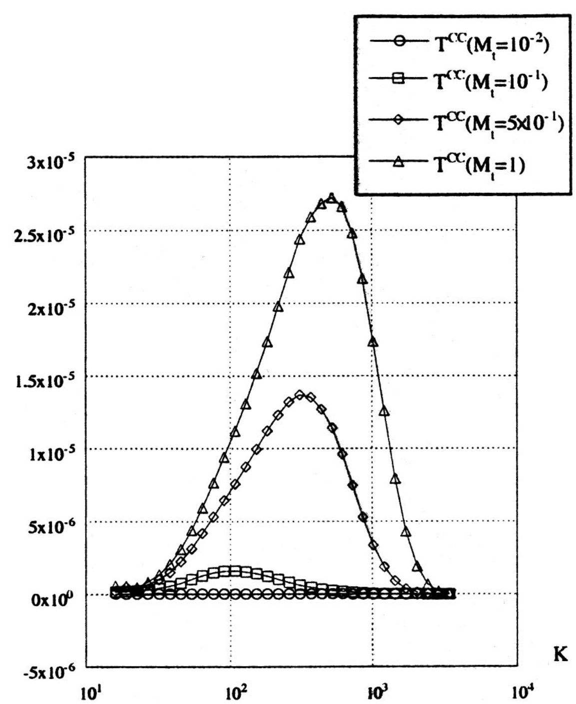

FIG. 12. Compressible transfer term at different M_t.

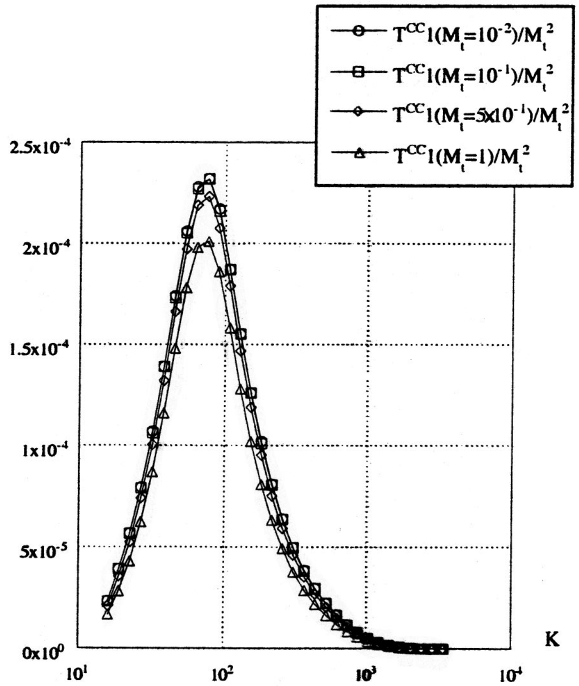

FIG. 13. T_1^{CC} divided by M_t^2.

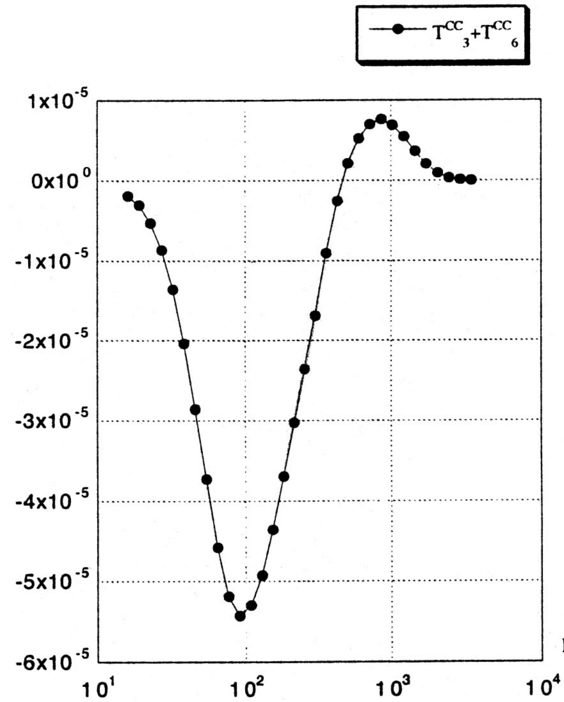

FIG. 15. $(T_3^{CC}+T_6^{CC})$ at different $M_t=1$.

tive, whereas the other term T_5^{CC} is always negative. In fact, the cancellations are so complete, the summation of these two terms is now actually negligible in comparison with T_1^{CC}. The physical explanation for this ''almost perfect'' cancelation is that T_2^{CC} and T_5^{CC} are the terms that take into account the interactions between slowly varying incompressible modes and two compressible modes (namely, the interactions between acoustic waves and a solenoidal field). This interaction results in the production of acoustic energy on the same wave number but now in another propagation direction. For an isotropic redistribution of acoustic energy, this effect of reorientation does not affect the spectral distribution of energy and leads to a zero net balance. Consequently, the most important term in the transport equation of E^{CC} is T_1^{CC}. This term is also much larger than all other contributors (T_3^{CC}, T_4^{CC}, and T_6^{CC}).

Comparing $T_5^{SS}(K)$ to $T_1^{CC}(K)$, it is clear that they have similar magnitude but with an opposite sign (Fig. 11). These two terms are essentially responsible of the energy exchanges between the solenoidal and compressible parts. Specifically, T_1^{CC} is the ''input'' energy term on the compressible mode whereas T_5^{SS} is the ''output'' term in the equation of the solenoidal mode. Based on these results obtained in spectral space, we conclude that there is a local transfer of energy from the solenoidal mode to the compressible mode. This result will be further confirmed by our analysis in the second part of the paper.

The total compressible transfer term T^{CC} is dependent on the compressibility (see Fig. 12). As expected, its magnitude increases with the increasing of turbulent Mach number. We note that there is a shift of the peak spectrum towards the

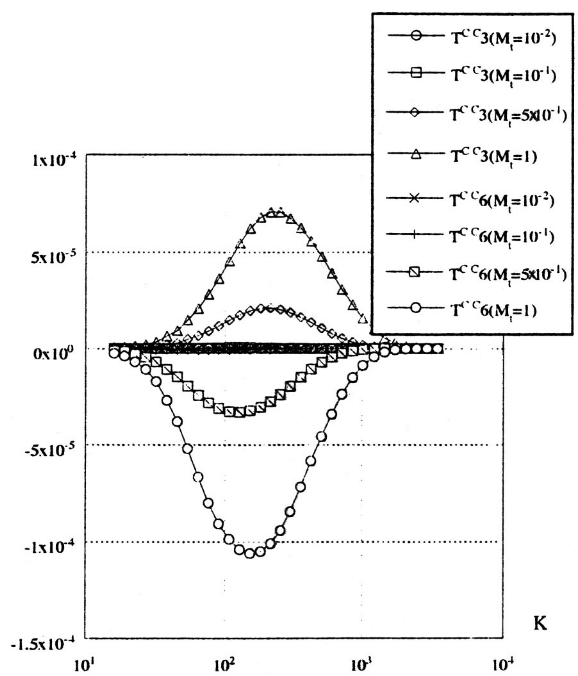

FIG. 14. T_3^{CC} and T_6^{CC} at different M_t.

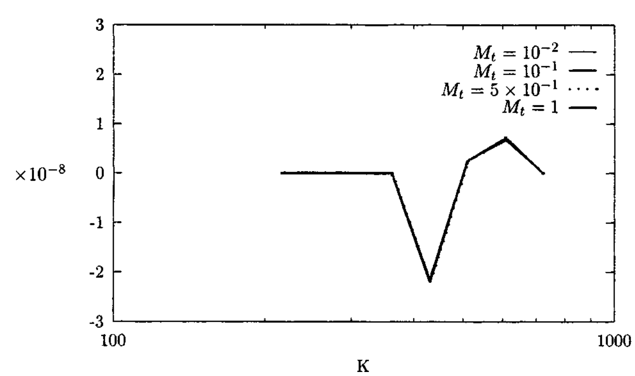

FIG. 16. Triadic solenoidal transfer term at different M_t.

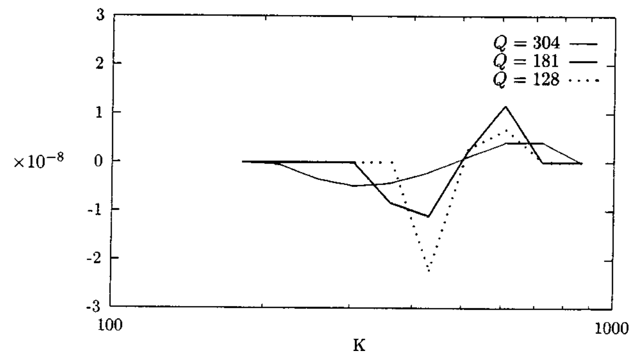

FIG. 17. Triadic solenoidal transfer term for different Q.

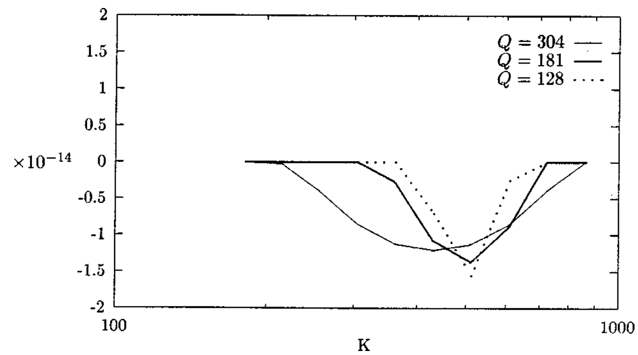

FIG. 19. Triadic T_5^{SS} for different Q.

large K. Based on the properties of $T_5^{SS}(K)$, we expect that its compressible counterpart $T_1^{CC}(K)$ should also scale as M_t^2. Since the spectra of T_1^{CC} divided by M_t^2 collapses (Fig. 13), this term can not be responsible for the peak shift of T^{CC}. Furthermore, the ''almost perfect'' cancellation between terms $T_2^{CC}(K)$ and $T_5^{CC}(K)$ for all turbulent Mach numbers is found. The term $T_4^{CC}(K)$ depends on the turbulent Mach number but its magnitude stays weak even at high M_t. Figure 14 shows the dependence of $T_3^{CC}(K)$ and $T_6^{CC}(K)$ on the values of turbulent Mach number. Although these two terms maintain opposite signs for all spectral space, the magnitudes and shape of $T_3^{CC}(K)$ and $T_6^{CC}(K)$ are clearly different. The ''imperfect'' cancellation between these two terms leads to a cascade type of compressible energy transfer (Fig. 15). These two terms involve $(E^{CC})^2$ and are important contributors at high turbulent Mach numbers. The interaction among the compressible mode begins to have influence. This cascade mechanism will be investigated in the next section.

IV. TRIADIC INTERACTIONS

The most fundamental building block of the energy transfer process is the triadic interactions. Specifically, we are interested in the energy transfer for a given mode **K** due to its interactions with all the pairs of modes **P** and **Q**=**K**−**P** that form a triangle with **K**. For this reason, we introduce the triadic energy transfer function, $T(K,P,Q)$, according to

$$T^{SS}(K)=\sum_{P,Q=|\mathbf{K}-\mathbf{P}|} T^{SS}(K,P,Q), \tag{27}$$

and

$$T^{CC}(K)=\sum_{P,Q=|\mathbf{K}-\mathbf{P}|} T^{CC}(K,P,Q). \tag{28}$$

Here $T(K,P,Q)$ is defined as energy transfer to K due to triads with one leg in Q and the other in P. The average procedure is performed over a spherical shell since the turbulence is isotropic.

An examination of the purely incompressible contributors $[T_S^{SS}(K,P,Q)]$ reproduces the results of incompressible turbulence (Domaradzki and Rogallo [18], Yeung, Brasseur, and co-workers [19,20], Ohkitani and Kida [21], Zhou [22], and Zhou, Yeung, and Brasseur [23]) and again indicates that the purely solenoidal triadic energy transfer is not affected by compressible effects. The triadic solenoidal transfer $T^{SS}(K,P,Q)$ (with the compressible terms included) is essentially the same for a wide range of turbulent Mach number values (Fig. 16). In the figures, the triadic transfer terms are given in $m^5\,s^{-3}$. Although Fig. 16 is only for $P=512$ and $Q=128$, we have examined other values of P and Q and found that our conclusion does not change with turbulent Mach number. As a result, the compressibility has very little influence on the solenoidal triadic interactions. Figure 17 is a typical plot showing how the structure of $T^{SS}(K,P,Q)$ changes with various Q values ($P=512$, $M_t=10^{-2}$). Again, this result is the same as that of incompressible turbulence.

We have found from the previous section that particular attention should be paid to the T_5^{SS} term since it is the term that is responsible for ''output'' energy from solenoidal to compressible mode. As the Mach number increases, the magnitude of the output energy increases but the basic structure remains (Fig. 18). Since this term represents an energy output at a localized spectral region, we refer to it as the

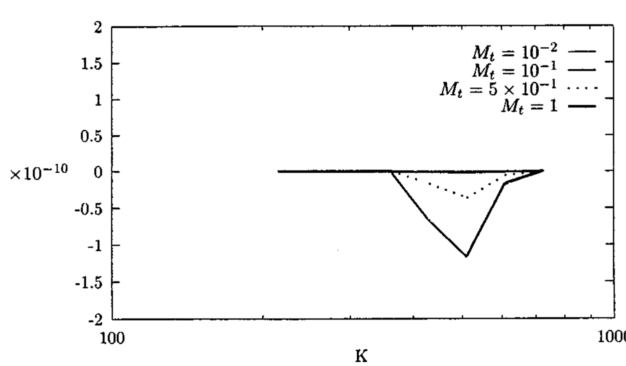

FIG. 18. Triadic T_5^{SS} at different M_t.

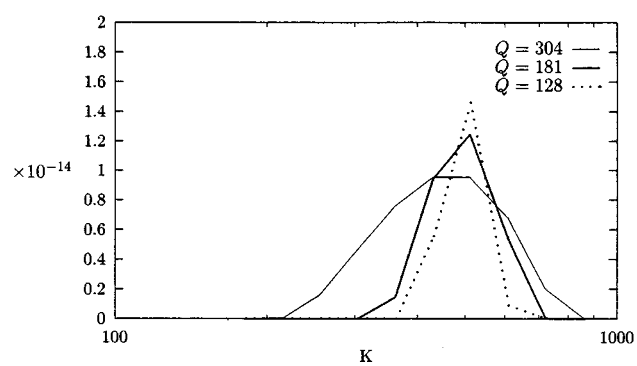

FIG. 20. Triadic compressible transfer term for different Q.

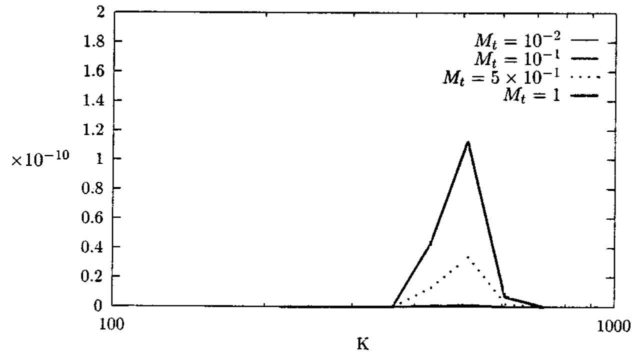

FIG. 21. Triadic T_1^{CC} at different M_t.

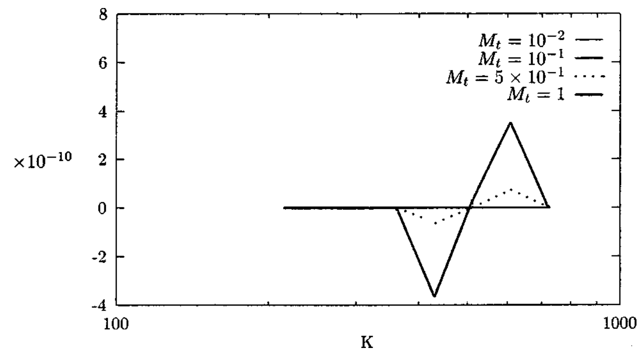

FIG. 23. Triadic $T_3^{CC}+T_6^{CC}$ at different M_t.

radiative (emission) triadic energy transfer. Figure 19 illustrates this type of interaction from various Q values at the given value of P ($P=512$) for $M_t=10^{-2}$. It is clear that the triadic interaction of this term is quite different from those of purely incompressible terms.

We now turn our attention to the triadic interactions in compressible energy transfer, $T^{CC}(K,P,Q)$. In Fig. 20, we present $T^{CC}(K,P,Q)$ for various Q values when P is in the inertial range ($P=512$). For this low Mach number 10^{-2}, we observe that the structures of $T^{CC}(K,P,Q)$ are rather similar for differing Q values. All of them show the *radiative* (absorption) type of energy transfer. We found that $T_1^{CC}(K,P,Q)$ is the dominant contributor to the compressible triadic energy transfer function. In fact, the absorption types of triadic energy transfer functions have the same magnitude as but opposite sign of those of the emission type (T_5^{SS} term). Figure 21 further demonstrates that the triadic interactions $T_1^{CC}(K,P,Q) \approx -T_5^{SS}(K,P,Q)$ for all turbulent Mach numbers are under consideration. As a result, we conclude that all compressible energy has been transferred locally (in spectral space) from the solenoidal component.

From the previous section, we have found that the sum of $T_3^{CC}(K,P,Q)+T_6^{CC}(K,P,Q)$ is an important contributor to the compressible energy transfer. At low Mach number ($M_t=10^{-2}$), terms $T_3^{CC}(K,P,Q)+T_6^{CC}(K,P,Q)$ are small and their triadic energy transfer terms show only a very weak energy cascade (Fig. 22). However, this situation changes rapidly as the Mach number increases. Indeed, the compressible energy cascade can be seen in Fig. 23 where $T_3^{CC}(K,P,Q)+T_6^{CC}(K,P,Q)$ are plotted for several higher Mach numbers. From this analysis, we conclude that at high Mach number the cascade of compressible turbulence is a direct result of the fact $T_3^{CC}(K,P,Q)+T_6^{CC}(K,P,Q) > T_1^{CC}(K,P,Q)$. To further demonstrate this point, we plot the total compressible energy transfer term, $T^{CC}(K,P,Q)$, at different Mach numbers. It is clear that $T^{CC}(K,P,Q)$ changes its characteristic features from radiative to cascade as the turbulent Mach number increases (Figs. 24 and 25). This is a result that can not be observed from the total compressible energy transfer function $T^{CC}(K)$. We can remark that for both the solenoidal and the compressible part, the shape of the transfer terms indicates an energy cascade process with a negative peak at left and a positive peak at right. In Fig. 16, we could see that the solenoidal triadic transfer becomes equal to zero when K is equal to the chosen P exactly ($P=512$). This feature corresponds to the one obtained in incompressible turbulence and indicates that the solenoidal triadic transfer is not influenced by the compressible effect. Concerning the compressible triadic transfer, we can see for high turbulent Mach number (Fig. 25) that the same picture appears and confirms again that an energetical cascade is present on the compressible mode.

V. CONCLUSION

We have investigated the energy transfer process of compressible turbulence using a two-point model, the EDQNM model. The velocity is decomposed into a compressible and solenoidal part. We especially studied the influence of the compressibility on the solenoidal and compressible mode,

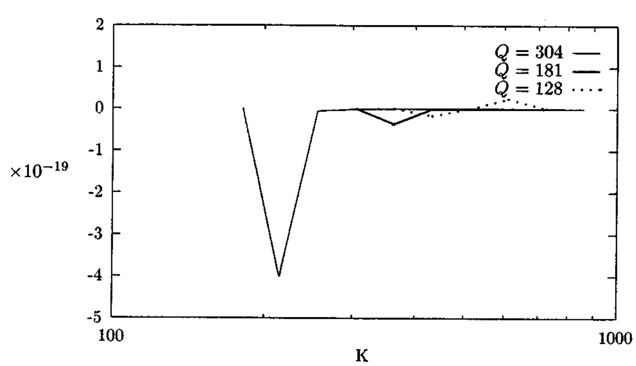

FIG. 22. Triadic $T_3^{CC}+T_6^{CC}$ for different Q.

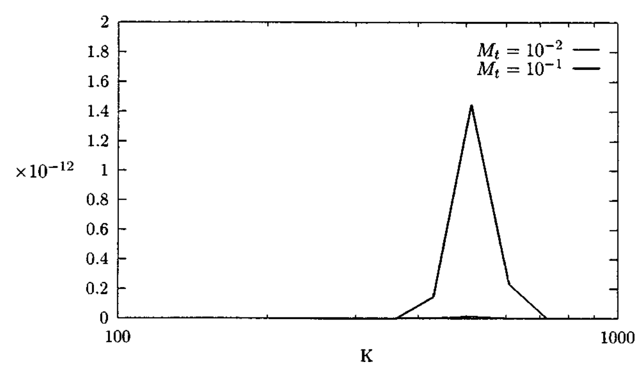

FIG. 24. Triadic T^{CC} at different low M_t.

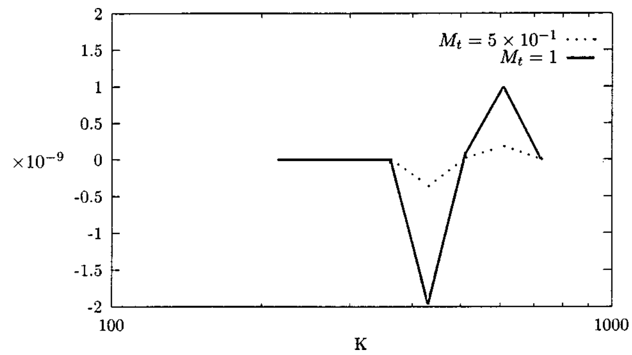

FIG. 25. Triadic T^{CC} at different high M_t.

the energy transfer between the two modes and the nature of the energy process on the compressible mode. An analysis of the energy transfer terms and of the solenoidal and compressible triadic interactions permitted to demonstrate that the compressible energy is locally transferred from the solenoidal part to the compressible part, for all the turbulent Mach numbers. We also observe that an energetical cascade appears on the compressible mode when the turbulent Mach number increases.

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